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Laminar film condensation on the outside of a vertical circular cylinder; including surface-tension effects

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Abstract

The paper presents a theoretical analysis of condensation of a saturated vapour on the outside of a vertical cylinder. Particular attention is focussed on the effect of including surface tension. The analysis is developed in terms of perturbation approximations and solutions obtained by perturbation theory are discussed in comparison with numerical solutions. The results show that surface tension plays a very significant role when the radius of the cylinder is small.

1. Introduction

The process of laminar film condensation is well documented in the literature because of the many important applications. There are, in particular, numerous papers devoted to problems which involve the formulation of a gravity-driven condensate layer. The first of these was presented by Nusselt [1] who analysed condensation onto a plane vertical wall, and it is relevant to acknowledge that most subsequent work has either served to establish the validity of Nusselt's assumptions and/or to extend these to other geometries or to include additional effects such as vapour superheating or the effect of non-condensables.

The case of condensation on the outside of a vertical circular cylinder was first investigated by Sparrow and Gregg [2]. There it was assumed that the thickness of the condensate layer was small compared with the radius of the cylinder and so the conventional boundary-layer approximation may be evoked.

Consider now the case the condensation on *thin* vertical cylinders. Two modifications of the Sparrow and Gregg [2] condensate flow are manifest. The first is that far downstream of the thermal leading edge the condensate thickness may be of the same order as the radius of the cylinder and so the above assumption of Sparrow and Gregg is no longer valid. The second is that near the thermal leading edge the vertical sectional radius of curvature of the condensate/vapour interface will be small and so surface tension effects will become important in this initial flow region. The aim of this paper is to quantify both of these initial and downstream effects on the condensate flow on vertical thin cylinders.

When the radius of the cylinder is much larger than the condensate thickness it is an easy matter to obtain reasonable approximations to the flow and heat-transfer characteristics by ignoring the vapour phase, except in so far as it is necessary to 'feed' the condensate interface. However, for thin cylinders the surface tension induces an important contribution to the interfacial stress tensor. For condensation in forced flows Beckett and Poots [3] showed how to include interfacial stresses via so-called 'thin-film' and 'thick-film' expansions. In particular it was shown that the 'thick-film' model was relevant for cases of appreciable condensation when the ratio $(T_S - T_W)/T_S = O(1)$, where T_S is the vapour saturation temperature and T_W is the wall temperature; when the ratio $(T_S - T_W)/T_S \ll 1$ the condensation rate is slight and the term 'thin-film' model is employed. In the case of the 'thick-film' model it is shown that the vapour phase behaves as a strong suction boundary layer. Subsequently Beckett, Hudson and Poots [4] considered condensation on a rotating disc. Since the terminology of 'thick-film' and 'thin-film' approximations already exists in the literature, it is appropriate to retain this nomenclature. It is, however, important to emphasise that the 'thick-film' model applies to films which are physically thin and the terminology is used to discuss the relative thickness when the two approximations apply. It may be useful to think of the 'thin-film' approximation as the 'very thin-film' approximation.

In this paper the 'thick-film' approximation is applied to condensation on the outside of vertical thin cylinders. The equations are cast in a form which is valid if the radius of the cylinder is arbitrarily small and accommodate the effects of surface tension.

2. The mathematical model

Attention is confined to steady-state condensation of a pure vapour at its saturation temperature T_S onto a vertical cylinder of radius a maintained at constant uniform temperature T_W, $T_W < T_S$. Although the condensed liquid flows down the outside of the cylinder the interfacial location will remain fixed; its surface will be denoted by $r = a + \delta(x)$ where $\delta(x)$ is the thickness of the condensate layer, x is a measure of distance down the cylinder and r the radial cylindrical polar variable. Referred to co-ordinates x and $y = r - a$, it can be shown that, providing the condensate film is thin, the flow and heat transfer in the condensate may be modelled by the boundary-layer equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{v}{a+y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x} + \nu\left[\frac{\partial^2 u}{\partial y^2} + \frac{1}{a+y}\frac{\partial u}{\partial y}\right] + g, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left[\frac{\partial^2 T}{\partial y^2} + \frac{1}{a+y}\frac{\partial T}{\partial y}\right], \tag{3}$$

where $u(x, y)$ and $v(x, y)$ are the velocity components in the x and y directions, $T(x, y)$ is the temperature distribution in the condensate, $p(x)$ is the pressure, ν is the kinematic viscosity and κ the thermal diffusivity, g is the gravitational constant and ρ is the density.

Similarly, if the radius of curvature in the flow direction is large in comparison with the condensate film thickness, then the equations for the vapour velocity components $u^*(x, y^*)$ and $v^*(x, y^*)$, where y^* is distance measured normal to the interface, may be written as

$$\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y^*} + \frac{v^*}{a+\delta+y^*} = 0, \tag{4}$$

$$u^*\frac{\partial u^*}{\partial x} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho^*}\frac{\mathrm{d}p^*}{\mathrm{d}x} + \nu^*\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{1}{a+\delta(x)+y^*}\frac{\partial u^*}{\partial y^*}\right) + g. \tag{5}$$

In the vapour phase the pressure, density and kinematic viscosity are denoted by p^*, ρ^* and ν^*, respectively, and elsewhere in the paper all starred quantities relate to the vapour phase.

The boundary conditions which are applied are as follows. At the cylinder surface, $y = 0$, we have

(i) the non-slip condition $u = v = 0$; (6)

(ii) specified temperature $T = T_W$. (7)

At the condensate/vapour interface, $y = \delta(x)$ or $y^* = 0$, we have

(iii) continuity in tangential velocity $u(x, \delta(x)) = u^*(x, 0)$; (8)

(iv) continuity of normal mass flow

$$\rho^*(v^*)_{y^*=0} = -\frac{\rho}{a+\delta}\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta}(a+y)u\,\mathrm{d}y; \tag{9}$$

(v) continuity of tangential shear stress

$$\mu\left(\frac{\partial u}{\partial y}\right)_{y=\delta(x)} = \mu^*\left(\frac{\partial u^*}{\partial y^*}\right)_{y^*=0}; \tag{10}$$

(vi) continuity of normal stress

$$p(x) - p^*(x) = \frac{\sigma}{a+\delta}, \tag{11}$$

where σ is the surface tension coefficient;

(vii) the liberation of latent heat

$$k\left(\frac{\partial T}{\partial y}\right)_{y=\delta(x)} = \rho\frac{h_{fg}}{a+\delta}\frac{\mathrm{d}}{\mathrm{d}x}\int_0^{\delta(x)}(a+y)u\,\mathrm{d}y, \tag{12}$$

where h_{fg} is the coefficient of latent heat.

At the outer edge of the vapour boundary layer

(viii) $u^*(x, y^*) \to 0$ as $y^* \to \infty$. (13)

Finally at the condensate/vapour interface

(ix) $T = T_S$ at $y = \delta(x)$. (14)

The particular forms of the conditions (8)–(13) are consequent on assuming $\mathrm{d}\delta/\mathrm{d}x \ll 1$, which is consistent with the earlier boundary-layer hypothesis. The arguments are very similar to those employed by Beckett and Poots [3], but a complete derivation may be found in Stead [5].

Before proceeding to the section describing the method of solution we note that since $p^*(x) = -\rho^* gx + C$ as $y^* \to \infty$ it is possible to remove $p(x)$ and $p^*(x)$ from equations (2) and (5) using (11) to yield

$$\frac{\mathrm{d}p^*}{\mathrm{d}x} = -\rho^* g \quad \text{and} \quad \frac{\mathrm{d}p}{\mathrm{d}x} = -\rho^* g - \frac{\sigma}{(a+\delta)^2}\frac{\mathrm{d}\delta}{\mathrm{d}x}. \tag{15}$$

3. Method of solution

It was shown by Beckett and Poots that there are two basic perturbation models for laminar film condensation problems, the so-called 'thin-film model' and the 'thick-film model'. Both involve condensate films which are physically thin and the nomenclature indicates the relative proportions; the 'thin-film model' is, however, only valid for situations in which the temperature difference $T_S - T_W$ is very small and for the vast majority of physical situations it is the 'thick-film model' which is appropriate. Indeed, the first term of the thick-film model gives the ad-hoc Nusselt solution for condensation onto a vertical plate.

The essence of the thick-film model is that the vapour flow behaves like a strong suction boundary layer and it becomes appropriate to construct the solution in terms of the, as yet, unknown suction velocity $V^*(x)$, where $V^*(x) = -v^*(x, 0)$. The method of Beckett and Poots [3] for solving the vapour phase follows similar lines to that used by Watson [6] for a single-phase strong-suction boundary layer. To first approximation the vapour momentum equation (5) reduces to

$$-V^*(x)\frac{\partial u^*}{\partial y^*} = \nu^*\left[\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{1}{a+\delta+y^*}\frac{\partial u^*}{\partial y^*}\right], \tag{16}$$

from which the tangential vapour velocity $u^*(x, y^*)$ satisfying the boundary condition (13) is

$$u^*(x, y^*) = A^*(x)\int_{y^*}^{\infty}\frac{\exp\{-V^*(x)y^*/\nu^*\}}{a+\delta+y^*}\,\mathrm{d}y^*, \tag{17}$$

where $A^*(x)$ is at this stage an arbitrary function of integration which is obtained along with the solution in the condensate phase.

Further analytical progress is possible if advantage is taken of the fact that the dimensionless parameter χ, where $\chi = C_p(T_S - T_W)/(\mathrm{Pr}\ h_{fg})$, is small for most engineering applications. Following the 'thick-film model' χ is used as a perturbation parameter for the condensate phase equations by writing $u(x, y) = \chi u_0(x, y) + O(\chi^2)$, $v(x, y) = \chi v_0(x, y) + O(\chi^2)$, $T(x, y) = T_0(x, y) + O(\chi)$.

The result is that the functions $u_0(x, y)$, $v_0(x, y)$ and $T_0(x, y)$ satisfy equations (2) and (3) without the nonlinear convective terms on the left-hand sides. These linear equations are easily solved to yield the following leading terms in the expansions for the temperature and velocity components:

$$T_0(x, y) = (T_S - T_W)\frac{\log(1+y/a)}{\log(1+\delta(x)/a)} + T_W, \tag{18}$$

$$u_0(x, y) = -G(x)\left\{\tfrac{1}{4}y^2 + \frac{ay}{2} - \frac{a^2}{2}\log(1+y/a)\right\} + A(x)\log(1+y/a). \tag{19}$$

Here

$$G(x) = \frac{g}{\nu}\left(a - \frac{\rho^*}{\rho}\right) + \frac{\sigma}{\mu(a+\delta)^2}\frac{\mathrm{d}\delta}{\mathrm{d}x}, \tag{20}$$

and $A(x)$ is another arbitrary function of integration, which is determined along with $A^*(x)$ by imposing the interfacial conditions; the conditions on $y = 0$ have been used to derive (18) and (19). Substituting (17) and (19) into the conditions (9) and (10) yields a pair of simultaneous equations for the functions $A(x)$ and $A^*(x)$ in terms of $\delta(x)$, and invoking the previous assumption that the suction velocity $V^*(x)$ is large, together with the fact that $\rho/\rho_s \gg 1$ for all condensation problems, * the expression for the velocity component $u_0(x, y)$, on elimination of $V^*(x)$, becomes

$$u_0(x, y) = -\left(\frac{g}{\nu} + \frac{\sigma}{\mu(a+\delta)^2}\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)\left[\frac{y^2}{4} + \frac{ay}{2} - \left(\frac{\delta^2}{4} + \frac{a\delta}{2}\right)\frac{\log(1+y/a)}{\log(1+\delta(x)/a)}\right]. \tag{21}$$

Next we can formulate an equation for $\delta(x)$ by substituting (18) and (20) into the latent-heat condition (12) at the interface. The algebraic manipulation is quite laborious, but some easing may be obtained by introducing the following non-dimensional quantities:

$$X = \frac{x}{a}, \quad \Delta = \frac{\delta}{a}, \quad \alpha = \frac{\sigma}{\rho g a^2}, \quad \Omega = \frac{\chi \nu^2}{g a^3}. \tag{22}$$

The eventual differential equation for $\Delta(X)$ is

$$\begin{aligned}
&\alpha\frac{\mathrm{d}^2\Delta}{\mathrm{d}X^2}\left\{\left(\frac{\Delta}{4} + \frac{5\Delta^2}{8} + \frac{5\Delta^3}{8} + \frac{5\Delta^4}{16} + \frac{\Delta^5}{16}\right)\log^2(1+\Delta) - \left(\frac{\Delta^2}{4} + \frac{\Delta^3}{2} + \frac{5\Delta^4}{16} + \frac{\Delta^5}{16}\right)\log(1+\Delta)\right\} \\
&+\alpha\left(\frac{\mathrm{d}\Delta}{\mathrm{d}X}\right)^2\left\{\left(\frac{1}{4} + \frac{\Delta}{2} + \frac{3\Delta^2}{4} + \frac{\Delta^3}{2} + \frac{\Delta^4}{8}\right)\log^2(1+\Delta) - \left(\frac{\Delta}{2} + \frac{3\Delta^2}{4} + \frac{\Delta^3}{4} + \frac{\Delta^4}{8}\right)\right. \\
&\qquad\qquad\left.\times\log(1+\Delta) + \frac{\Delta^2}{4} + \frac{\Delta^3}{4} + \frac{\Delta^4}{16}\right\} \\
&+\frac{\mathrm{d}\Delta}{\mathrm{d}X}\left\{\left(\frac{1}{4} + \frac{3\Delta}{2} + \frac{15\Delta^2}{4} + 5\Delta^3 + \frac{15\Delta^4}{4} + \frac{3\Delta^5}{2} + \frac{\Delta^6}{4}\right)\log^2(1+\Delta)\right. \\
&\qquad -\left(\frac{\Delta}{2} + \frac{9\Delta^2}{4} + 4\Delta^3 + \frac{7\Delta^4}{2} + \frac{3\Delta^5}{2} + \frac{\Delta^6}{4}\right)\log(1+\Delta) \\
&\qquad \left.+\left(\frac{\Delta^2}{4} + \frac{3\Delta^3}{4} + \frac{13\Delta^4}{4} + \frac{3\Delta^5}{8} + \frac{\Delta^6}{16}\right)\right\} \\
&= \Omega(1+\Delta)^3\log(1+\Delta),
\end{aligned} \tag{23}$$

for which the appropriate boundary conditions are

$$\Delta(0) = 0 \quad \text{and } \Delta(X, \alpha) \to \Delta(X, 0) \quad \text{as } X \to \infty. \tag{24}$$

The first condition simply states that the film commences at the top of the cylinder while the second expresses the fact that as $X \to \infty$ the film thickness will grow to an extent that the circumferential curvature will tend to zero.

It is at this stage where we can either choose to include or ignore the surface tension. Putting $\alpha = 0$ reduces the order of equation (23) so that only one condition, i.e. $\Delta(0) = 0$, is necessary.

* It is inappropriate here to give the rigorous derivation of the 'thick-film' approximation as already developed in [3] and [4]. There it is shown that the above expansions are valid provided $(T_S - T_W)/T_S = O(1)$, $\chi \ll 1$ and $\lambda\chi \gg 1$, where $\lambda = (\nu_s/\nu_s^*)^{1/2}(\rho_s/\rho_s^*)$. Note that these inequalities are satisfied by most engineering situations.

For small values of X it is a straight forward, but laborious, procedure to assume a series expansion for $\Delta(X)$ in terms of X which results in the following expression

$$\Delta(X) = 2(\Omega X)^{1/4} - \tfrac{2}{15}(\Omega X)^{1/2} - \tfrac{4}{675}(\Omega X)^{3/4} - \tfrac{1244}{10125}\Omega X + O(X^{5/4}). \tag{25}$$

Clearly this is of limited use and for a solution which is valid as $X \to \infty$ we need an alternative approach. The linear equation which results from putting $\alpha = 0$ in equation (23) is written in integral form as

$$\Omega X = \int \left\{ \left(\frac{\Delta^3}{4} + \frac{3\Delta^2}{4} + \frac{3\Delta}{4} + \frac{1}{4} \right) \log(1+\Delta) - \left(\frac{\Delta^3}{4} + \frac{3\Delta^2}{4} + \frac{\Delta}{2} \right) + \left(\frac{\Delta^4}{16} + \frac{\Delta^3}{4} + \frac{\Delta^2}{4} \right) \Big/ [(1+\Delta)\log(1+\Delta)] \right\} \mathrm{d}\Delta + \text{constant} \tag{26}$$

and by introducing a new variable W by

$$W = \log(1+\Delta), \tag{27}$$

the asymptotic form as $W \to \infty$ becomes

$$\Omega X = \frac{\mathrm{e}^{4W}}{16}\left(W - \frac{5}{4} + \frac{1}{4W} + O\left(\frac{1}{W^2}\right)\right) + O(\mathrm{e}^{-2W}), \tag{28}$$

and finally

$$1 + \Delta(X) = \exp\left\{ \frac{\beta}{4} - \frac{1}{4}\log\frac{\beta}{4} + \frac{1}{4\beta}\left(5 + \log\frac{\beta}{4}\right) + \frac{1}{32\beta^2}\left(43 - 12\log\frac{\beta}{4} + 4\log^2\frac{\beta}{4}\right) + O\left(\frac{1}{\beta^3}\right)\right\} \tag{29}$$

where $\beta = \log(16\Omega X)$.

Having completed the solution for $\alpha = 0$ we now return to the second-order equation (23) which applies when the surface tension is non-zero. Again we begin by seeking a series solution for $\Delta(X)$ which is valid for small values of X. An order of magnitude analysis of (23) indicates that $\Delta(X) \approx X^{2/5}$ for small values of X and on assuming a series in powers of $X^{2/5}$ there results the following expression:

$$\Delta(X) = \left(\frac{50\Omega}{\alpha}\right)^{1/5} X^{2/5} + \frac{9}{28}\left(\frac{50\Omega}{\alpha}\right)^{2/5} X^{4/5} - \frac{5}{12\alpha}X - \frac{232}{33075}\left(\frac{50\Omega}{\alpha}\right)^{3/5} X^{6/5} - \frac{467}{616\alpha}\left(\frac{50\Omega}{\alpha}\right)^{1/5} X^{7/5} + O(X^{8/5}). \tag{30}$$

As previously stated the effects of surface tension die away as $X \to \infty$, hence there is no need to seek a further approximation because the asymptotic solution which was developed for the case $\alpha = 0$ will also apply when $\alpha \neq 0$; that is, the series solution (29) also applies as $X \to \infty$ for $\alpha \neq 0$. For both cases, $\alpha = 0$ and $\alpha \neq 0$, it is necessary to generate numerical solutions for the intermediate region; these numerical solutions are also valuable in assessing the accuracy and validity of the series solutions. The numerical solution is easily obtained using Runge-Kutta integration; the series for small X enable the solution to be initiated away from the leading edge singularity.

4. Discussion of results

Qualitatively the effect of including surface tension is obvious from the small X solutions (25) and (30). That is, the film thickness $\delta(x)$ initially grows at a rate proportional to $x^{1/4}$ if surface tension is neglected, but proportional to $x^{2/5}$ if surface tension is included – the former is consistent with Sparrow and Gregg's model for cylinders of appreciable radii.

The quantitative effect of the surface tension is made for the cases of steam condensing onto three cylinders of radii 0.5, 0.1 and 0.05 cm, respectively, the physical data for these cases are given in Table 1. The comparative rates of growth of condensate film thickness are displayed in Tables 2–4; these show marked differences in the initial development of thickness but asymptotic developments in which the role of surface tension becomes insignificant. (The tabulated data are derived from solutions, using Runge-Kutta integration, of equation (23) for the dimensionless film thickness $\Delta(X)$). The domain for X for which the results are presented

Table 1. Property values for water at $T_S - T_W = 100\,°C$.

h_{fg} (cal/g)	538.83		
ρ (g/cm^3)	0.9578		
C_p (cal/g °C)	1.0075		
ν (cm^2/s)	2.936×10^{-3}		
Pr	1.7370		
σ (dyne/cm)	58.8		
	$a = 0.5$ cm	$a = 0.2$ cm	$a = 0.05$ cm
$\Omega = \dfrac{C_p(T_S - T_W)\nu^2}{\Pr h_{fg} g a^3}$	7.57×10^{-9}	1.18×10^{-7}	7.57×10^{-6}
$\alpha = \dfrac{\sigma}{\rho g a^2}$	0.25	1.56	25.03

Table 2. The effect of surface tension on the condensate film thickness $\Delta(X)$ for pure water vapour condensing on a cylinder of radius = 0.5 cm.

X	Surface tension included $\alpha = 0.25$		Surface tension ignored $\alpha = 0$	
	Series (29)	Numerical	Series (24)	Numerical
0.00002	0.087×10^{-2}	0.087×10^{-2}	0.125×10^{-2}	0.125×10^{-2}
0.00005	0.122	0.123	0.157	0.164
0.0001	0.156	0.157	0.187	0.191
0.0005	0.244	0.265	0.279	0.280
0.001	0.265	0.323	0.332	0.333
0.005	−0.194	0.493	0.496	0.496
0.01	−0.608	0.588	0.590	0.590
0.05		0.881	0.882	0.882
0.1		1.048	1.049	1.049
0.5		1.568	1.568	1.568
1.0		1.864	1.864	1.864

Table 3. Condensate film thickness $\Delta(X)$ for the cases of zero and non-zero surface tension when $a = 0.2$ cm.

X	$\alpha = 1.56$	$\alpha = 0$
0.00002	0.108×10^{-2}	0.274×10^{-2}
0.00005	0.156	0.326
0.0001	0.204	0.380
0.0005	0.381	0.557
0.001	0.495	0.661
0.005	0.875	0.986
0.01	1.094	1.173
0.05	1.728	1.753
0.1	2.070	2.084
0.5	3.112	3.115
0.75	3.445	3.447
1.0	3.701	3.704

is restricted to the range over which this effect is established. In the case of the cylinder of radius 0.5 cm, Table 2 shows inclusion of the surface tension reduces the condensate film thickness near $X = 0$ by approximately 44%, and there is no significant difference in film thickness when $X = 0.5$. For the cylinders of small radius the effect of including surface tension is even more marked, the increases in film thickness near $X = 0$ are approximately 154% and 390% for the cylinders of radii 0.2 and 0.05 cm, respectively. Also the length of the cylinders over which there is a difference between the thickness predicted with and without surface tension increases as the cylinder radius decreases.

Table 2 also confirms the validity, near $X = 0$, of the perturbation solutions in terms of X to the differential equation for $\Delta(X)$. Moreover it can be seen that the radius of convergence of the expansion for the equation when $\alpha \neq 0$, that is, when surface tension is included, is much smaller than when $\alpha = 0$. Indeed when surface tension is excluded the perturbation expansion (25) gives three-figure agreement for the film thickness over the domain displayed. Although an asymptotic series, namely (29), has been formulated it is not compared with numerical solutions because in order to include terms such as $\log(\log(16X\Omega))$ it would have been necessary to have X of the order of 10^8.

Finally it is interesting to note the effects of including surface tension on the heat flux at the surface of the cylinder. Introducing the Nusselt number $\mathrm{Nu}(X)$ by

$$\mathrm{Nu}(X) = \frac{a\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_S - T_W)}, \tag{31}$$

Table 4. Condensate film thickness $\Delta(X)$ for zero and non-zero surface tension when $a = 0.05$ cm.

X	$\alpha = 25.03$	$\alpha = 0$
0.00002	0.143×10^{-2}	0.701×10^{-2}
0.00005	0.207	0.922
0.0001	0.273	1.074
0.0005	0.519	1.576
0.001	0.685	1.869
0.005	1.302	2.788
0.01	1.714	3.314
0.05	3.226	4.952
0.1	4.291	5.887
0.5	7.601	8.795
1.0	9.594	10.456
4.0	14.47	14.76

Table 5. Comparison of the Nusselt number for cylinders of radii 0.5 cm, 0.2 cm and 0.05 cm with and without surface tension.

X	$a = 0.5$ cm		$a = 0.2$ cm		$a = 0.05$ cm	
	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 1.56$	$\alpha = 0$	$\alpha = 25.05$	$\alpha = 0$
0.00002	1.147×10^3	0.802×10^3	0.925×10^3	0.365×10^3	0.698×10^3	0.129×10^3
0.00005	0.815	0.610	0.644	0.307	0.484	0.108
0.0001	0.637	0.525	0.490	0.264	0.367	0.093
0.0005	0.378	0.357	0.263	0.180	0.193	0.064
0.001	0.310	0.301	0.203	0.152	0.147	0.054
0.005	0.203	0.202	0.115	0.102	0.077	0.036
0.01	0.171	0.170	0.092	0.088	0.059	0.030
0.05	0.114	0.114	0.058	0.058	0.032	0.020
0.1	0.096	0.096	0.049	0.049	0.024	0.017
0.5	–	–	–	–	0.014	0.011
1.0	0.054	0.054	0.028	0.028	0.011	0.010
4.0	–	–	–	–	0.007	0.007

it follows that in terms of $\Delta(X)$ this can be written as

$$\mathrm{Nu}(X) = \frac{1}{\log(1 + \Delta(X))}. \tag{32}$$

Table 5 shows the dramatic increase in heat transfer which typically follows from inclusion of the surface tension into the problem.

5. Conclusion

The inclusion of surface tension is an important feature in the modelling of film condensation onto vertical cylinders, particularly if the radius of the cylinder is small. It has been shown that the importance is greatest near the top of the cylinder, but as the film thickness increases down the cylinder the surface-tension effects disappear. The results seem to confirm general intuitive ideas but the quantitative increase in heat transfer is perhaps rather more surprising.

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